Year 12 Mathematics: Specialist
Term 2 2020

Test 2 *Calculator Assumed*
Functions, Graphs & Vectors in 3D

Student Name: ______________________________

Teacher: Alfonsi Moore

Working Time: 25 minutes
Formula Sheet provided. *1 A4 page of notes (2 sided) allowed.*

*Attempt **all** questions.*
*All necessary working and reasoning must be shown for **full marks**.*

Total Marks
$\overline{}$ 29

Question 6. **(7 marks)**

The vectors in $\mathbb{R}^3$, $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{w}$, have been defined below.

$$\boldsymbol{u} = 3\boldsymbol{i} - 5\boldsymbol{j} + \boldsymbol{k} \qquad \boldsymbol{v} = -\boldsymbol{i} + 10\boldsymbol{j} - 8\boldsymbol{k} \qquad \boldsymbol{w} = 5\boldsymbol{i} - 2\boldsymbol{k}$$

Determine:

(a) $\boldsymbol{u} + \boldsymbol{v} - 2\boldsymbol{w}$ (1 mark)

(b) $|3\boldsymbol{u} + 2\boldsymbol{v}|$, leaving your answer exact. (1 mark)

(c) $(-2\boldsymbol{w}) \cdot (-\boldsymbol{u})$ (1 mark)

(d) $\left(\frac{1}{2}\boldsymbol{v}\right) \times \boldsymbol{w}$ (1 mark)

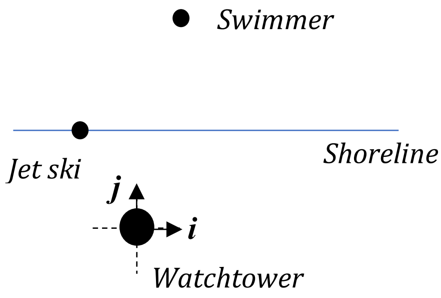
For the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$,

(e) Determine the angle between the two vectors, rounded to the nearest degree. (1 mark)

(f) State whether the scalar projection of $\boldsymbol{u}$ onto $\boldsymbol{v}$ would be positive or negative. Justify your answer. (2 marks)

Question 7. **(8 marks)**

From the position of a surf-lifesaving watchtower, a swimmer is spotted caught in a rip with a relative position vector of $\boldsymbol{r}_S = 55\boldsymbol{i} + 400\boldsymbol{j}$ metres.



The rip pulls the swimmer out to sea with a velocity of $\boldsymbol{v}_S = -0.2\boldsymbol{i} + \boldsymbol{j}$ metres per second.

At the instant the swimmer is first spotted, the order is given for a surf-lifesaving jet ski to leave from the shoreline with a relative position of $\boldsymbol{r}_J = -60\boldsymbol{i} + 150\boldsymbol{j}$ metres from the watchtower.

The velocity of the jet ski, taking into account the current, is $\boldsymbol{v}_J = 7\boldsymbol{i} + 16\boldsymbol{j}$ metres per second.

(a) Show that the jet ski will not collide with the swimmer. (3 marks)

(b) *Assuming both the swimmer and jet ski remain on the same paths with the same velocities,* determine:

(i) the time taken for the jet ski to get to the point that is closest to the swimmer and state this minimum distance. (3 marks)

(ii) the position vectors of the swimmer and jet ski at this closest approach. (2 marks)

Question 8. **(14 marks)**

A line, plane and a sphere are defined by the following vector equations.

$$L : \boldsymbol{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix} \qquad \Pi : \boldsymbol{r} \cdot \begin{pmatrix} 1 \\ 4 \\ -6 \end{pmatrix} = 2 \qquad S : \left| \boldsymbol{r} - \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right| = 4$$

For the following questions, round all final answers to two decimal places.

(a) Explain why the line must intersect the sphere at two points and determine the position vectors of the two points of intersection. (4 marks)

(b) Determine the acute angle between the line and the plane. (2 marks)

(c) Determine the position vector of the point of intersection between the line and the plane. (3 marks)

Recall the equations of the plane and sphere below.

$$\Pi : \boldsymbol{r} \cdot \begin{pmatrix} 1 \\ 4 \\ -6 \end{pmatrix} = 2 \qquad\qquad S : \left| \boldsymbol{r} - \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right| = 4$$

(d) Determine whether or not the plane intersects the sphere.

If so, determine the radius of the intersection circle formed by the plane and the sphere.

If not, determine the shortest distance from the plane to the sphere.

(5 marks)

End of Calculator Assumed Section